Pattern Recognition Problems

1. For the following image $A_{1,45^0}$, compute the co occurrence matrix. From the co occurrence matrix, compute the texture features entropy, and inverse element difference moment.

$$\begin{bmatrix} 3 & 5 & 6 & 7 & 2 & 4 & 3 & 1 \\ 5 & 6 & 7 & 5 & 8 & 7 & 1 & 0 \\ 0 & 6 & 3 & 4 & 2 & 6 & 5 & 3 \\ 3 & 2 & 6 & 5 & 2 & 4 & 7 & 3 \\ 0 & 4 & 6 & 5 & 7 & 3 & 2 & 1 \\ 2 & 3 & 6 & 2 & 4 & 7 & 0 & 2 \\ 5 & 3 & 4 & 7 & 3 & 2 & 0 & 2 \\ 4 & 7 & 5 & 2 & 1 & 0 & 5 & 3 \end{bmatrix}$$

2. Assume a shape parameter $\theta = \dfrac{Area}{Perimeter^2}$ used for shape classification. Comment on the maximum and minimum values of θ.

3. Assume that the 3-dimensional feature vectors belonging to two classes ω_1 and ω_2 follow gaussian distribution having he following parameters.

$$\begin{array}{ll} i.\ C_1 = \{x_1, x_2\} & C_2 = \{x_3, x_4\} \\ ii.\ C_1 = \{x_1, x_4\} & C_2 = \{x_2, x_3\} \\ iii.\ C_1 = \{x_1, x_2, x_3\} & C_2 = \{x_4\} \end{array}$$

$$\mu_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \quad \Sigma_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 2 \\ 0 & 2 & 5 \end{bmatrix}$$

$$\mu_2 = \begin{bmatrix} 9 \\ 3 \\ 7 \end{bmatrix} \quad \Sigma_2 = \begin{bmatrix} 4 & 2 & 3 \\ 2 & 5 & 3 \\ 3 & 3 & 4 \end{bmatrix}$$

Find out the decision boundary and classify the feature vector $[4 \quad 9 \quad 2]^t$.

4. Given the following observations of x: 44, 60, 17, 20, 72, 30, 35, 49, 50, 58, 26, 29, 70, 74, 91, 61, 32, 44, 75, 15, 12, 53, 27, 84, 81, 90, 88, 72, 55, 53, 21, 25, 56, 52, 51, 49 and 75.

   Make the density histogram of values using a reasonable interval size.

5. Following sets of 2-D feature vectors from classes A and B are given.

$$\left\{\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}1\\3\end{pmatrix},\begin{pmatrix}2\\1\end{pmatrix},\begin{pmatrix}2\\1\end{pmatrix},\begin{pmatrix}2\\2\end{pmatrix},\begin{pmatrix}3\\3\end{pmatrix},\begin{pmatrix}4\\1\end{pmatrix},\begin{pmatrix}4\\2\end{pmatrix}\right\} \in A$$
$$\left\{\begin{pmatrix}3\\2\end{pmatrix},\begin{pmatrix}4\\3\end{pmatrix},\begin{pmatrix}4\\3\end{pmatrix},\begin{pmatrix}5\\3\end{pmatrix},\begin{pmatrix}4\\4\end{pmatrix},\begin{pmatrix}6\\3\end{pmatrix},\begin{pmatrix}4\\6\end{pmatrix},\begin{pmatrix}7\\3\end{pmatrix}\right\} \in B$$

   Using rectangular window of size 3x3, compute $p((3.5,3)^t \mid A)$ and $p((3.5,3)^t \mid B)$. Classify $(3.5,3)^t$ if $P(A) = \frac{1}{3}$ and $P(B) = \frac{2}{3}$.

6. Consider the following set of seven 2-dimensional feature vectors:

$$X_1 = (1,0)^t, X_2 = (0,1)^t, X_3 = (0,-1)^t, X_4 = (0,0)^t, X_5 = (0,2)^t,$$
$$X_6 = (0,-2)^t, X_7 = (-2,0)^t$$

   If $X_1, X_2, X_3 \in \omega_1$ and $X_4, X_5, X_6, X_7 \in \omega_2$, sketch the decision boundary resulting from the nearest neighbor rule.

7. Obtain the generalized discriminant functions for a C class problem assuming multivariate normal distribution of feature vectors.

8. The following sets of points are given for a two class problem.

$$\left\{\begin{pmatrix}2\\6\end{pmatrix},\begin{pmatrix}3\\4\end{pmatrix},\begin{pmatrix}3\\8\end{pmatrix},\begin{pmatrix}4\\6\end{pmatrix}\right\} \in \omega_1$$
$$\left\{\begin{pmatrix}3\\0\end{pmatrix},\begin{pmatrix}1\\-2\end{pmatrix},\begin{pmatrix}5\\-2\end{pmatrix},\begin{pmatrix}3\\-4\end{pmatrix}\right\} \in \omega_2$$

   Assuming that the feature vectors in every class follow Gaussian distribution and the classes are equally probable, determine the decision boundary between the two classes.

9. Consider a two class problem having three independent binary features x_i, i=1,2,3. Given $P(\omega_1) = P(\omega_2) = 0.5$ and $p_i = \text{prob}[x_i = 1|\omega_1] = 0.8$ and $q_i = \text{prob}[x_i = 1|\omega_2] = 0.5$ (i=1,2,3), sketch the decision boundary between the two classes.

10. The following sets of points are given for a two class problem

$$(1,6)^t, (2,8)^t, (3,3)^t, (5,6)^t \in \omega_1$$
$$(6,3)^t, (8,2)^t, (9,5)^t, (11,3)^t \in \omega_2$$

Assuming that the feature vectors in every class follow Gaussian distribution and the classes are equally probable, and the loss matrix ($\lambda_{i,j}$) as given below, classify the feature vector $(8,7)^t$ such that the risk involved is minimum.

$$\lambda_{i,j} = \begin{bmatrix} 0 & 0.7 \\ 0.5 & 0 \end{bmatrix}$$

11. To reduce the dimensionality of feature vectors in pattern classification problems the d-dimensional feature vectors are projected onto a $\hat{d}$-dimensional space where $\hat{d} << d$. Show that the error due to such projection is minimized if $\hat{d}$-dimensional space is same as the eigen space of the feature vectors.

12. The sets of points belonging to two different classes are as given below.

$$(1,3)^t, (4,2)^t, (2,7)^t, (8,11)^t \in \omega_1$$
$$(7,4)^t, (11,5)^t, (12,12)^t, (15,10)^t \in \omega_2$$

The feature vectors are projected onto a line to represent each feature vector by a single feature. Find out the best direction of the line of projection that maintains the separability between the two classes.

13. In a four category classification problem the following samples are given along with their class belongingness.

$$\left\{(1,1)^t, (1,2)^t, (2,1)^t, (2,2)^t\right\} \in \omega_1$$
$$\left\{(1,4)^t, (1,5)^t, (2,4)^t, (2,5)^t\right\} \in \omega_2$$
$$\left\{(4,1)^t, (4,2)^t, (5,1)^t, (5,2)^t\right\} \in \omega_3$$
$$\left\{(4,4)^t, (4,5)^t, (5,4)^t, (5,5)^t\right\} \in \omega_4$$

Following Kesler's construction compose the training samples for classifier design.

14. A Hopfield network is to remember the following patterns.

A = + + - - + +
B = + - + - + +
C = + + + - - -

Compute the node interconnection weights for this network.

15. Design a multilayer feedforward neural network with two input layer nodes and one output layer node that accepts 2-dimensional binary feature vectors such that the output will be 0 if both the components of the input vector are same and output will be 1 otherwise.

16. Let $x_1 = (4,5)^t$, $x_2 = (1,4)^t$, $x_3 = (0,1)^t$, $x_4 = (5,0)^t$. Determine which of the following gives best clustering in sum-of squared error sense.

$i.\ C_1 = \{x_1, x_2\} \qquad C_2 = \{x_3, x_4\}$
$ii.\ C_1 = \{x_1, x_4\} \qquad C_2 = \{x_2, x_3\}$
$iii.\ C_1 = \{x_1, x_2, x_3\} \qquad C_2 = \{x_4\}$

17. Cluster the following set of points using similarity matrix.

(2,2), (3,1),(3,2), (2,3), (3,3), (3,5), (4,3), (4,5), (5,5), (7,2), (7,3)

Assume nodes A and B to be connected only when the Euclidean distance between them is less than 1.2.

18. A set of samples is given in the following table. Perform hierarchical clustering of the samples using complete-linkage algorithm and Euclidean distance. Show the distance matrices and the dendogram.

Sample	x	y
1	0.0	0.0
2	0.5	0.0
3	0.0	2.0
4	2.0	2.0
5	2.5	8.0
6	6.0	3.0
7	7.0	3.0

19. A hidden Markov Model θ with four hidden states ω_i, (i = 0, 1, 2, 3) and five visible symbols v_j, (j = 0, 1, 2, 3, 4) is specified with the following state transition probabilities a_{ij} and symbol emission probabilities b_{jk}.

$$a_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.2 & 0.3 & 0.1 & 0.4 \\ 0.2 & 0.5 & 0.2 & 0.1 \\ 0.8 & 0.1 & 0.0 & 0.1 \end{bmatrix} \qquad b_{jk} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.4 & 0.1 & 0.2 \\ 0 & 0.1 & 0.1 & 0.7 & 0.1 \\ 0 & 0.5 & 0.2 & 0.1 & 0.2 \end{bmatrix}$$

Find out the probability that the model θ generates the visible symbol sequence

$$V^5 = (v_1, v_3, v_4, v_2, v_0)$$

Assume ω_1 to be the initial state and ω_0 to be the absorbing state.

20. The following set of 2-D feature vectors from two classes A and B are used to train a Fuzzy Min-Max neural network with compensatory neuron. The feature vectors are presented to the neural network in the given sequence of serial numbers. Assuming that hyperboxes in any dimension can not grow beyond 0.2, answer the following.
  (a) Design the Fuzzy Min-Max neural network with compensatory network and show it diagrammatically.
  (b) Draw the corresponding hyperboxes showing clearly the respective min points and the max points.

Sl. No.	Feature Vector	Class
1	0.9, 0.9	A
2	0.8, 0.8	A
3	0.5, 0.73	B
4	0.35, 0.6	B
5	0.4, 0.5	B
6	0.2, 0.3	B
7	0.7, 0.7	A
8	0.6, 0.8	A

Sl. No.	Feature Vector	Class
9	0.45, 0.65	A
10	0.35, 0.38	A
11	0.75, 0.5	B
12	0.6, 0.3	B
13	0.25, 0.2	A
14	0.85, 0.35	A
15	0.65, 0.2	A

  (c) Using the above neural network, classify the feature vectors (0.55, 0.6) and (0.73, 0.32). Assume the fuzzyness parameter $\gamma = 10$.